Measuring real consumption and consumer price index bias under lockdown conditions

W. Erwin Diewert
University of British Columbia, UNSW Sydney and NBER

Kevin J. Fox
UNSW Sydney

Abstract. Millions of goods and services are now unavailable in many countries due to the current coronavirus pandemic, dramatically impacting on the construction of key economic statistics used for informing policy. This situation is unprecedented; hence, methods to address it have not previously been developed. Current advice to national statistical offices from the International Monetary Fund, Eurostat and the United Nations is shown to result in downward bias in the consumer price index (CPI) and upward bias in real consumption. We conclude that, to produce a meaningful CPI within the lockdown period, it is necessary to establish a continuous consumer expenditure survey.

Résumé. Mesure de la consommation réelle et des biais par défaut de l'indice des prix à la consommation en période de confinement. En raison de la pandémie actuelle de COVID-19, des millions de biens et de services sont présentement indisponibles dans de nombreux pays, entravant significativement l'élaboration de statistiques économiques clés visant à éclairer les politiques. Cette situation est sans précédent, et aucune méthode n'a encore été développée pour y remédier. Les avis dispensés aux organismes nationaux de statistiques par le Fonds monétaire international, Eurostat et les Nations Unies se traduisent par des biais systématiques par défaut quant à l'indice des prix à la consommation (IPC), et par des biais par excès quant à la consommation réelle. Notre conclusion est que pour produire un IPC significatif lors d'un confinement, il est nécessaire de procéder à une enquête continue sur les dépenses de consommation.

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Corresponding author: Kevin Fox, K.Fox@unsw.edu.au

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> The CPI is a critical input to economic policy making, particularly during periods of economic uncertainty.
>
> — International Monetary Fund (2020)

1. Introduction

The coronavirus pandemic has led to widespread lockdowns of non-essential retail outlets and many service industries such as regular restaurant services, tourism activities, sporting events, gyms, international air travel and most types of personal services.[1] There are significant consequences of the sudden unavailability of many consumer goods and services for the measurement of inflation and real consumption.

A lockdown of economic activity means that there is a massive *disappearing products problem*. Countries base the construction of their consumer price index (CPI), the main index of inflation, on a fixed basket of goods that people typically buy. But with lockdowns, the fixed basket becomes almost totally irrelevant as many of those typical items are no longer available; see Cavallo (2020), Carvalho et al. (2021) and Dunn et al. (2020) for evidence of dramatically changing consumer expenditure patterns arising from the pandemic. If consumer prices cannot be measured accurately, then real consumption cannot be measured accurately either. Policy-makers and the public will eventually lose confidence in such indexes. This creates a crisis for policy and business decisions that rely heavily on these statistics.

Advice to national statistical offices (NSOs) from the International Monetary Fund, Eurostat and the United Nations for dealing with this problem is to simply implement a carry-forward methodology. That is, the price for a commodity for the period prior to the lockdown is used (with some adjustment for inflation) when an item is missing; see International Monetary Fund (2020), Eurostat (2020), UNECE (2020) and the summary in online technical appendix A. With the missing prices restored, index number construction can continue as before with the fixed consumption basket. We show that following this advice leads to a downward bias in estimating changes in the cost of living and an upward bias in estimating changes in real consumption.

With a lockdown, we have the opposite of the *new products problem*: a product is available for purchase in the current period but was not available in the previous period. We take the reservation price approach for the treatment of new products, which was developed by Hicks (1940, p. 114) and adapt it to cover the case of disappearing products following Hofsten (1952, pp. 95–97).[2] *Reservation prices* are those that correspond to zero demand for the products.

1 The term “lockdown” is used to indicate a spectrum of restrictions that have different effects on the availability of products and the associated impacts on consumer behaviour.

2 For related materials on modeling new and disappearing goods bias, see Hausman (1996, 1999, 2003), de Haan and Krsinich (2012, 2014) and Diewert et al. (2017).

The unavailability of goods and services under a lockdown will lead to a substantial drop in welfare.[3] Using the economic approach to index number theory in order to measure these declines in welfare, it is likely that the reservation prices for unavailable products will have to be much greater than the corresponding prices in the previous period. This approach allows us to identify biases from the methods currently being recommended by international agencies.

Our conclusions for constructing the most informative prices indexes in this context can be summarized as follows:

1. In the short run, collect whatever prices are available and supplement these from scanner data and web scraped prices to make up for missing prices due to changes in price collection methodology.[4] For prices that are still missing, use inflation adjusted carry forward prices, consistent with current advice from the international agencies.
2. At the same time, put in motion some method for getting *current expenditure weights* for the consumption basket. This would require either a continuous consumer expenditure survey or the use of new sources of data. These new sources could include credit card companies and companies that produce household expenditure data from households scanning barcodes of purchased items ("Homescan" data).
3. Once the new consumer expenditure information becomes available, produce a new analytic CPI. This would be revisable while the new methodology was developed further. It would supplement the existing CPI, which would likely be heavily compromised due to the treatment of missing prices and use of out-of-date expenditure weights.

For what follows, it is important to explain why the Fisher price index is preferred to the Laspeyres or Paasche price indexes. The Laspeyres price index prices out the basket of goods and services that is consumed in the *base period* at base period prices and at current period prices. The Laspeyres price index P_L is the ratio of these two costs using the prices of the current period in the

3 A referee notes the following: "Experiences of countries like Venezuela show that the reduction in availability of products is not limited to just pandemics, and there are potentially large welfare implications to products disappearing. When there are regulatory or political barriers to product availability, this reduces real consumption by a greater amount than would be indicated by a standard price indexes. Applying reservation price arguments suggests that the real decline in consumption may be greatly understated if standard price index measures are applied."

4 Some NSOs already make extensive use of scanner data from retail chains and are relatively well placed in this regard; see, e.g., Australian Bureau of Statistics (2017). Problems remain, however, for broader categories of goods and services.

numerator and the prices of the base period in the denominator. The Paasche price index prices out the basket of goods and services that is consumed in the *current period* at base period prices and at current period prices. The Paasche price index, P_P, is the ratio of these two costs using the prices of the current period in the numerator and the prices of the base period in the denominator. Both indexes are equally plausible and intuitively easy to understand. But the Laspeyres index will tend to show a greater increase in prices than the corresponding Paasche index.

Good statistical practice suggests that when one has two equally plausible measures of the same phenomenon, it is better to take an average of the two measures to give a single measure of the phenomenon. The geometric average of the Laspeyres and Paasche price indexes is a useful average of P_L and P_P that is equal to the Fisher price index. The Fisher index satisfies the important time reversal test (and many other tests) so that inflation measured going forward is the reciprocal of inflation going backwards. The good test performance of the Fisher index explains why taking the geometric average of P_L and P_P is preferred rather than taking some other form of average of P_L and P_P.[5] Thus, the paper will focus on how to measure *lockdown bias*, which we define to be the difference between a Laspeyres or Lowe (or fixed basket) CPI and the Fisher price index.

As a general longer-term goal, statistical agencies need to move away from a fixed basket Lowe index and attempt to produce approximations to Laspeyres and Paasche indexes and hence Fisher indexes. This will allow consumer price inflation to be measured by an index that is relevant to current consumer expenditure patterns.

The paper is organized as follows. In section 2, we study how the sudden unavailability of many goods and services affects the measurement of real consumption, that is, *quantity indexes* for household consumption. It may seem odd that in a paper focused mainly on potential bias in a CPI that we first look at the problems associated with the measurement of real consumption (and by extension, with the measurement of real GDP). However, construction of a consumer price index goes hand in hand with the construction of a corresponding measure of real consumption. In the end, economists and governments are concerned with the *welfare implications* of the pandemic on real consumption. It proves to be convenient to look at the problems associated with the measurement of real consumption during lockdown conditions before we look at the associated CPI measurement problems. In sections 3 and 4, our focus shifts to consumer price indexes. Section 3 looks at comparisons between the Laspeyres, Paasche and Fisher price indexes under pandemic conditions, while section 4 examines the properties of a fixed basket or Lowe price index, which is used in construction of the CPI in many countries. Section 5 concludes.

5 See Diewert (1992) for a listing of the tests that the Fisher price index satisfies.

Three online appendices accompany this article. Appendix A provides a survey of the methods suggested by various statistical agencies. Appendix B specializes the algebra explained for the many commodity case in the main text to the case of only two commodities. The advantage of this simplification is that the various concepts can be illustrated in a simple diagram. Appendix C provides some theoretical guidance on how to construct generalized reservation prices when some market consumer goods and services are provided by the government or businesses to the household at zero cost (or at a highly subsidized prices) as responses to the pandemic. For example, how should a temporary rent holiday be treated in a CPI? The reservation price for the household in this situation is equal to its willingness to pay for this good or service and the Appendix indicates how in theory this price could be estimated using econometric techniques.

2. Measuring real consumption when transitioning to a lockdown economy

Suppose that the period prior to the lockdown is period 0 and the first lockdown period is the subsequent period, 1. The period of time could be a month or a quarter. We initially assume that data on prices and quantities that are in scope for the CPI are available to the statistical agency. We divide the CPI goods and services into two groups: Group 1 prices and quantities are available in periods 0 and 1 and Group 2 prices and quantities are available only in period 0.[6] Denote the period t price and quantity vectors for Group 1 products by $p^t \equiv [p_1^t, ..., p_M^t] >> 0_M$ and $q^t \equiv [q_1^t, ..., q_M^t] >> 0_M$ for $t = 0,1$.[7] The Group 2 price and quantity vectors for period 0 are $P^0 \equiv [P_1^0, ..., P_N^0] >> 0_N$ and $Q^0 \equiv [Q_1^0, ..., Q_N^0] >> 0_N$. We take the Group 2 quantity vector for period 1 to be a vector of zero components, i.e., we assume that

$$Q^1 \equiv 0_N. \tag{1}$$

It is not clear how to define the period 1 price vector P^1 for the products that are not available in period 1. In these notes, we will apply the *economic approach to index number theory* and assume that the appropriate definition for the missing prices are the relevant *Hicksian reservation prices*, i.e., $P^{1*} \equiv [P_1^{1*}, ..., P_N^{1*}] >> 0_N$, where $P_n^{1*} > 0$ is the price for commodity n

6 The methodology to be developed can be applied to a subset of the CPI, i.e., to an elementary category. However, the category has to be broad enough so that it contains some continuing commodities or products and some commodities that have disappeared due to the lockdown. The algebra that follows assumes that information on (unit value) prices and the corresponding quantities are available for the commodities in scope during period 0.

7 Notation: $p^t >> 0_M$ (0_M) means that all components of the M dimensional vector p^t are positive (nonnegative). The inner product of the vectors p^t and q^t is defined as $p^t \cdot q^t \equiv \sum_{m=1}^{M} p_m^t q_m^t$.

that will cause the corresponding period 1 consumer demand $Q_n^{\ 1}$ to equal 0 for that commodity for n = 1, ..., N.

In most cases, it is safe to assume that the reservation price for product n in period 1, $P_n^{\ 1*}$, will be considerably larger than the actual price for product n in period 0, $P_n^{\ 0}$, i.e., it is reasonable to assume that[8]

$$P^{1*} \gg P^0. \tag{2}$$

It is important to understand why the reservation price for a product that is suddenly unavailable is likely to be greater than the prevailing price in the prior period when it was available. In the most extreme form of lockdown, individuals are confined to their houses or apartments. If they live in a small apartment, the resulting confinement is very similar to being in prison. Individuals are typically willing to pay large sums of money to avoid going to prison (think of high lawyer fees). Going to jail or being subject to a strict lockdown decreases welfare to a very substantial degree. A way of measuring the decrease in welfare is to use the reservation price concept. In order to get a substantial drop in welfare due to a lockdown, the reservation prices for unavailable goods and services will have to be much greater than the corresponding prices in the previous period.

Before we look at the implications of our assumptions for the construction of a consumer price index going from period 0 to 1, we look at the implications for the measurement of real consumption. The *"true" overall Laspeyres quantity index*, Q_L, is defined as follows:[9]

$$\begin{aligned} Q_L &\equiv [p^0 \cdot q^1 + P^0 \cdot Q^1]/[p^0 \cdot q^0 + P^0 \cdot Q^0] \\ &= p^0 \cdot q^1/[p^0 \cdot q^0 + P^0 \cdot Q^0] \text{ using assumption (1)} \\ &= [p^0 \cdot q^1/p^0 \cdot q^0]\{p^0 \cdot q^0/[p^0 \cdot q^0 + P^0 \cdot Q^0]\} \\ &= Q_{Lq} s_q^{\ 0}, \end{aligned} \tag{3}$$

where the *Laspeyres quantity index for always present commodities*, Q_{Lq}, is defined as

$$Q_{Lq} \equiv p^0 \cdot q^1/p^0 \cdot q^0. \tag{4}$$

The *period 0 expenditure share of always present commodities* is $s_q^{\ 0}$, defined as

8 In technical appendix B, we assume M = 1 and N = 1. Using the economic approach to index number theory, it is possible to show that it must be the case that $P^{1*} \geq (p^1/p^0)P^0$, where p^0 and P^0 are the observed period 0 prices for the two commodities, p^1 is the period 1 price for the continuing commodity and P^{1*} is the period 1 reservation price vector for the unavailable commodity. P^{1*} will equal $(p^1/p^0)P^0$ only if the two commodities are perfect substitutes. Note that $(p^1/p^0)P^0$ is the inflation adjusted carry forward price vector for the unavailable commodity. See technical appendix C for information on how to define reservation prices.

9 Note that the reservation prices P^{1*} do not enter the definition for Q_L.

$$s_q{}^0 \equiv p^0 \cdot q^0/[p^0 \cdot q^0 + P^0 \cdot Q^0]. \tag{5}$$

Thus, overall Laspeyres real consumption growth, Q_L, is equal to Laspeyres consumption growth of always available commodities Q_{Lq} times the share of always available commodities in period 0, $s_q{}^0$. Note that Q_L, Q_{Lq} and $s_q{}^0$ depend on observable data and, in principle, can be constructed by the statistical agency. This is not the case for the Paasche quantity index.

The *"true" overall Paasche quantity index*, $Q_P{}^*$, is defined as follows:[10]

$$\begin{aligned}
Q_{P^*} &\equiv [p^1 \cdot q^1 + P^{1*} \cdot Q^1]/[p^1 \cdot q^0 + P^{1*} \cdot Q^0] \\
&= p^1 \cdot q^1/[p^1 \cdot q^0 + P^{1*} \cdot Q^0] \text{ using assumption (1)} \\
&= [p^1 \cdot q^1/p^0 \cdot q^0]p^0 \cdot q^0/[(p^1 \cdot q^0/p^0 \cdot q^0)p^0 \cdot q^0 + (P^{1*} \cdot Q^0/P^0 \cdot Q^0)P^0 \cdot Q^0] \\
&= [p^1 \cdot q^1/p^0 \cdot q^0]s_q{}^0/P_{Lq}s_q{}^0 + P_{LQ}{}^*s_Q{}^0] \\
&= P_{Lq}Q_{Pq}s_q{}^0/\left[P_{Lq}s_q{}^0 + P_{LQ}{}^*s_Q{}^0\right] \text{ given } p^1 \cdot q^1/p^0 \cdot q^0 = P_{Lq}Q_{Pq} \\
&= Q_{Pq}s_q{}^0P_{Lq}/\left[P_{Lq}s_q{}^0 + P_{LQ}{}^*s_Q{}^0\right],
\end{aligned} \tag{6}$$

where the *Paasche quantity index for always available commodities*, Q_{Pq}, is defined as

$$Q_{Pq} \equiv p^1 \cdot q^1/p^1 \cdot q^0. \tag{7}$$

The *Laspeyres price indexes for the always available and unavailable commodities*, P_{Lq} and $P_{LQ}{}^*$, respectively, are defined as follows:

$$P_{Lq} \equiv p^1 \cdot q^0/p^0 \cdot q^0, \tag{8}$$

$$P_{LQ^*} \equiv P^{1*} \cdot Q^0/P^0 \cdot Q^0. \tag{9}$$

The *period 0 expenditure shares of the available and unavailable commodities* are $s_q{}^0$ and $s_Q{}^0$, defined as

$$s_q{}^0 \equiv p^0 \cdot q^0/[p^0 \cdot q^0 + P^0 \cdot Q^0], \tag{10}$$

10 Note that the reservation prices P^{1*} do enter the definition for Q_P: the bigger is the gap between $P^{1*} \cdot Q^0$ and $P^0 \cdot Q^0$, the smaller $Q_P{}^*$ becomes. Thus in order to compute real consumption (using either the test or economic approaches to index number theory), it is absolutely necessary to compute the Paasche quantity index (or an approximation to it) in addition to the Laspeyres quantity index, which misses the effect of higher reservation prices in period 1. The Fisher quantity index will usually provide an adequate approximation to the change in welfare due to the lockdown; see Diewert (1976, 2021a).

$$s_Q{}^0 \equiv P^0 \cdot Q^0/[p^0 \cdot q^0 + P^0 \cdot Q^0]. \tag{11}$$

Note that definition (9) for the Laspeyres index for the unavailable commodities, $P_{LQ}{}^* \equiv P^{1*}\cdot Q^0/P^0\cdot Q^0$, depends on the vector of period 1 reservation prices P^{1*} for these products.[11] Thus formula (6) for the true overall Paasche quantity index, $Q_P{}^* = Q_{Pq}s_q{}^0P_{Lq}/[P_{Lq}s_q{}^0 + P_{LQ}{}^*s_Q{}^0]$ would seem to be of only theoretical interest. However, it is possible to provide some very rough estimates for P^{1*}.

Suppose that the Laspeyres inflation for unavailable products was equal to Laspeyres inflation for the continuing products; i.e., suppose that $P^{1*}\cdot Q^0/P^0\cdot Q^0 = p^1\cdot q^0/p^0\cdot q^0$. This means that the reservation prices P^{1*} are such that:

$$P_{LQ}{}^* = P_{Lq}. \tag{12}$$

Assumption (12) sets Laspeyres type inflation $P_{LQ}{}^*$ for the unavailable goods and services equal to the actual Laspeyres inflation for the always available products P_{Lq}. This is consistent with a form of *inflation adjusted carry forward pricing* for unavailable products, i.e., if we set $P^{1*} = P_{Lq}P^0 = (p^1\cdot q^0/p^0\cdot q^0)P^0$, then assumption (12) will be satisfied.[12] Under this assumption, define the corresponding overall Approximate Paasche quantity index, $Q_P{}^A$, as follows:[13]

$$\begin{aligned} Q_P{}^A &\equiv Q_{Pq}s_q{}^0P_{Lq}/\left[P_{Lq}s_q{}^0 + P_{Lq}s_Q{}^0\right] \text{ using definition (6) and assumption (12)} \\ &= Q_{Pq}s_q{}^0 \\ &> Q_{Pq}s_q{}^0P_{Lq}/\left[P_{Lq}s_q{}^0 + P^*_{LQ}s_Q{}^0\right] \text{ if } P_{LQ}{}^* > P_{Lq} \\ &= Q_P{}^*, \end{aligned} \tag{13}$$

given the period 0 expenditure shares sum to 1, i.e., we have $s_q{}^0 + s_Q{}^0 = 1$.

11 The fact that the true Laspeyres price index, $P_{LQ}{}^*$, for unavailable commodities has an asterisk superscript is meant to alert the reader that the index depends on the reservation prices for the unavailable products, which are not directly observable. Because the true overall Paasche quantity index, $Q_P{}^*$, depends on $P_{LQ}{}^*$, we have added an asterisk superscript to Q_P as well.

12 From technical appendix B, it can be seen that in general, these inflation adjusted carry forward reservation prices, $P_{Lq}P^0$, will be too low from the perspective of the economic approach to index number theory.

13 The superscript A was added to Q_P to indicate that the resulting overall Paasche quantity index is only an approximation to the true Paasche quantity index, Q_P. However, $Q_P{}^A$ does not depend on the unobservable reservation prices P^{1*} and so $Q_P{}^A$ is a practical approximation to Q_P.

The relationships in (13) show that the practical approximation to the true Paasche quantity index defined by $Q_P{}^A$ is equal to the Paasche quantity index for continuing commodities, Q_{Pq}, times the expenditure share of continuing commodities in the base period $s_q{}^0$. The implication of the inequality in (13) is that the practical Paasche index $Q_P{}^A$ has an *upward bias* relative to the true Paasche quantity index $Q_P{}^*$ provided that the true Laspeyres price index for unavailable commodities $P_{LQ}{}^*$ is greater than the Laspeyres price index for continuing commodities P_{Lq}.[14] For many countries experiencing massive shutdowns of various industries, it is almost certain that $P_{LQ}{}^*$ is very much bigger than P_{Lq} so that the upward bias in $Q_P{}^A$ relative to Q_P is likely to be very large indeed.

Note the similarity of expressions (3) and (13) for the Laspeyres and approximate Paasche quantity indexes. Under assumption (12), we obtain the following simple formula for the *Approximate Fisher quantity index*, $Q_F{}^A$:

$$\begin{aligned} Q_F{}^A &\equiv \left[Q_L Q_P{}^A\right]^{1/2} \\ &= \left[Q_{Lq} s_q{}^0 Q_{Pq} s_q{}^0\right]^{1/2} \text{ using (13)} \\ &= s_q{}^0 \left[Q_{Lq} Q_{Pq}\right]^{1/2} \\ &= s_q{}^0 Q_{Fq}, \end{aligned} \tag{14}$$

where $Q_{Fq} \equiv [Q_{Lq}Q_{Pq}]^{1/2}$ is the Fisher quantity index defined over only available commodities where Q_{Lq} is defined by (4) and Q_{Pq} is defined by (7). Thus under assumption (12), real consumption going from period 0 to 1 can be measured by the Fisher quantity index Q_{Fq} defined over only always available commodities times the expenditure share on always available commodities in period 0.[15]

However, as indicated above, it is very likely that the reservation prices P^{1*} are very much higher than their period 0 counterpart prices P^0 and hence the implicit inflation for the unavailable commodities will likely be much greater than the observed inflation in the always available commodities. As was the case for the inequality in (13), we make the following assumption:

$$P_{LQ}{}^* > P_{Lq}. \tag{15}$$

14 Alternatively, $Q_P{}^A > Q_P{}^*$ if the true reservation prices $P^{1*} > P_{Lq}P^0$, where $P_{Lq}P^0 = (p^1q^0/p^0q^0)P^0$ are the inflation adjusted carry forward prices for unavailable commodities in period 1.

15 This result is similar to the inflation adjusted carry forward price methodology explained in Triplett (2004), de Haan and Krsinich (2012, 2014) and Diewert et al. (2017).

Using assumption (15) and the inequality in (13) ($Q_P{}^A > Q_P{}^*$), we see that the true overall Paasche quantity index, $Q_P{}^*$, (which may be difficult to accurately calculate due to a lack of information on the reservation prices of the unavailable products P^{1*}) will be less than the share adjusted Paasche quantity index calculated using only the always available commodities, $Q_{Pq}s_q{}^0$ (which can be calculated). Thus under the reasonable assumption (15), we have the following relationships between the true overall Laspeyres, Paasche and Fisher measures of real consumption, Q_L, $Q_P{}^*$ and $Q_F{}^* \equiv [Q_L Q_P{}^*]^{1/2}$, and their counterpart subindexes that use only information on the prices and quantities of available products, Q_{Lq}, Q_{Pq} and Q_{Fq}:

$$Q_L = Q_{Lq}s_q{}^0, \tag{16}$$

$$Q_P^* < Q_{Pq}s_q{}^0, \tag{17}$$

$$Q_F^* < Q_{Fq}s_q{}^0. \tag{18}$$

Thus, if assumption (15) holds, the true overall Paasche and Fisher quantity indexes, $Q_P{}^*$ and $Q_F{}^*$, will be *less* than their share adjusted counterpart indexes, $Q_{Pq}s_q{}^0$ and $Q_{Fq}s_q{}^0$, which do not require information on reservation prices. The size of the gaps in the inequalities (17) and (18) will grow as the size of the gap in the inequality (15) grows.[16] It is very likely that the gaps in the inequalities (15), (17) and (18) are substantial for many countries that have implemented significant lockdowns of economic activity.[17]

At the time of writing, it has become clear that the COVID-19 virus has changed the preferences of many households. Households are reluctant to shop for goods and services in person and they are reluctant to offer labour services at unsafe workplaces. This fact affects the above analysis in at least two ways:

- Government-mandated shutdowns of many industries are only part of the lockdown story; many business establishments will shut down due to safety concerns for both their customers and workers. Thus, the lockdown effects on output and consumption are bigger than just government mandated shutdowns of business and household activities. The products that correspond to the vector Q include not only products that have disappeared due to government orders but also products that households no longer wish to purchase due to safety concerns.
- The reservation prices that appear in the above algebra (and in appendix B) are reservation prices based on the preferences that prevailed before the lockdowns took place. For the post-lockdown preferences, the

16 Of course, the size of the gap in (17) will always be larger than the size of the gap in (18).

17 See technical appendix B for some rough and ready methodology that will enable the reader to form an approximation to the gap in (18).

reservation prices for the products in the Q vector are essentially infinite. Basically, changing preferences mean that a new CPI series along with a new real consumption series needs to be constructed for the lockdown period.[18]

Once the economy has stabilized under lockdown conditions, it is appropriate to use chained Fisher indexes using only commodities that are available under shutdown conditions; see section 6 of Diewert and Fox (2020) for more detail on this recommendation. If the degree of shutdown increases from month to month, the analysis in the present section can again be applied to the smaller set of available commodities as we go from month to month. However, when the lockdown ends in period $\tau > 1$, instead of linking period τ to period $\tau - 1$, linking period τ to period 0 will minimize any potential chain drift problems. The basic idea here is that the quantity vector $[q^\tau, Q^\tau]$ is likely to be more similar to $[q^0, Q^0]$ than to the quantity vectors that pertain to the intervening lockdown periods.

3. Constructing a cost of living index when transitioning to a lockdown economy

The *true overall Paasche price index*, P_P, is defined as follows:[19]

$$\begin{aligned} P_P &\equiv [p^1 \cdot q^1 + P^{1*} \cdot Q^1]/p^0 \cdot q^1 + P^0 \cdot Q^1] \\ &= p^1 \cdot q^1/p^0 \cdot q \text{ using assumption (1)}, \quad Q^1 = 0_N \\ &\equiv P_{Pq,} \end{aligned} \tag{19}$$

where P_{Pq} is the Paasche price index that uses only the price and quantity data pertaining to the always available products in the two periods being compared. The above equality tells us that the restricted domain Paasche

18 Suppose household preferences in the pre-lockdown period can be represented by the utility function, $F(f^1(q),f^2(Q))$, where all three functions are increasing, linearly homogeneous and concave functions. Then using pre-lockdown preferences, real consumption growth going from period 0 to 1 is equal to $F(f^1(q^1), f^2(0_N))/F(f^1(q^0),f^2(Q^0))$. Post-lockdown preferences set $Q = 0_N$, and using these preferences, real consumption growth going from period 0 to 1 can be set equal to $F(f^1(q^1),f^2(0_N))/F(f^1(q^0),f^2(0_N))$ or, more simply, to $f^1(q^1)/f^1(q^0)$. Thus these two real consumption series going from period 0 to 1 are not really comparable. If period t is a lockdown period, then real consumption growth relative to period 0 can be represented by $f^1(q^t)/f^1(q^0)$. If a subsequent period τ is a "back to normal" period, then we can measure consumption growth relative to period 0 using period 0 preferences as $F(f^1(q^\tau),f^2(Q^\tau))/F(f^1(q^0),f^2(Q^0))$.

19 Note that the true Paasche index P_P does not depend on the vector of period 1 reservation prices P^{1*}. Thus P_P is likely to be very much lower than the true Laspeyres index, $P_L{}^*$, to be defined shortly.

price index, P_{Pq}, is equal to the true overall Paasche price index, P_P. Thus the overall Paasche price index can be constructed without the use of imputed price data for period 1 prices, P^{1*}.

The *overall true Laspeyres price index*, $P_L{}^*$, is defined as follows:[20]

$$\begin{aligned} P_L{}^* &\equiv [p^1 \cdot q^0 + P^{1*} \cdot Q^0]/[p^0 \cdot q^0 + P^0 \cdot Q^0] \\ &= [(p^1 \cdot q^0/p^0 \cdot q^0)p^0 \cdot q^0 + (P^{1*} \cdot Q^0/P^0 \cdot Q^0)P^0 \cdot Q^0]/[p^0 \cdot q^0 + P^0 \cdot Q^0] \\ &= P_{Lq}s_q{}^0 + P_{LQ^*}s_Q{}^0 \\ &= P_{Lq}\{s_q{}^0 + [(P^{1*} \cdot Q^0/P^0 \cdot Q^0)/(p^1 \cdot q^0/p^0 \cdot q^0)]s_Q{}^0\}, \end{aligned} \tag{20}$$

where the period 0 expenditure shares, $s_q{}^0$ and $s_Q{}^0$, were defined by (10) and (11), the Laspeyres price index for continuing products, P_{Lq}, was defined by (8) and the Laspeyres price index for unavailable products, $P_{LQ}{}^*$, was defined by (9).[21] The problem with definition (20) is that the vector of period 1 reservation prices, P^{1*}, is not directly observable, and, hence, the overall Laspeyres index, $P_L{}^*$, and the "true" Laspeyres index for unavailable commodities in period 1, $P_{LQ}{}^*$, cannot be readily calculated.

When a commodity is temporarily out of stock in a retail outlet, many statistical agencies simply *carry forward* the observed (unit value) price for the product from the previous period and use this carry forward price in place of the Hicksian reservation prices that were used above in our index number calculations. Thus define the overall *Laspeyres price index* $P_L{}^C$ *using* (*inflation unadjusted*) *carry forward prices*, P^0, in place of the reservation prices, P^{1*}, as follows:

$$\begin{aligned} P_L{}^C &\equiv [p^1 \cdot q^0 + P^0 \cdot Q^0]/[p^0 \cdot q^0 + P^0 \cdot Q^0] \\ &= [(p^1 \cdot q^0/p^0 \cdot q^0)p^0 \cdot q^0 + P^0 \cdot Q^0]/[p^0 \cdot q^0 + P^0 \cdot Q^0] \\ &= P_{Lq}s_q{}^0 + s_Q{}^0, \end{aligned} \tag{21}$$

where the period 0 expenditure shares, $s_q{}^0$, and $s_Q{}^0$, were defined by (10) and (11) and the Laspeyres price index for continuing products, P_{Lq}, was defined by (8).

Using expressions (20) and (21), it can be seen that the following inequality holds between the simple unadjusted carry forward Laspeyres index, P_L^C, and the true Laspeyres index, $P_L{}^*$:

$$P_L{}^* > P_L^C \text{ if and only if } P_{LQ}{}^* > 1. \tag{22}$$

20 Again we add a superscript asterisk to P_L to indicate that the true overall Laspeyres price index requires a knowledge of the period 1 reservation prices for unavailable commodities, P^{1*}.

21 Compare (20) with the simpler expression defined by (B35) in technical appendix B.

As in the previous section, it is reasonable to assume that reservation prices P^{1*} are considerably larger than prices P^0 that prevailed prior to lockdown conditions. Hence we can safely assume that the true Laspeyres subindex for unavailable commodities, $P_{LQ}{}^* \equiv P^{1*}{\cdot}Q^0/P^0{\cdot}Q^0$, is greater than 1 and thus $P_L{}^C$ will have a *substantial downward bias* compared to the true Laspeyres price index $P_L{}^*$ that uses reservation prices.

Following Triplett (2004) and many other authors,[22] statistical offices often multiply the prior period prices that are not available in the present period by the price inflation of related commodities. We will use either the Laspeyres or Paasche price index for continuing commodities to do this indexation and compare the resulting overall Laspeyres price indexes to $P_L{}^C$.

Define the *inflation adjusted carry forward prices* P^{1L} using the Laspeyres price index for continuing commodities, P_{Lq}, as the inflation adjusting index as follows:

$$P^{1L} \equiv P_{Lq}P^0 = (p^1 \cdot q^0/p^0 \cdot q^0)P^0. \tag{23}$$

Define the Laspeyres index using the inflation adjusted carry forward prices, P^{1L}, in place of the true prices, P^{1*}, as $P_L{}^{CL}$:

$$\begin{aligned} P_L{}^{CL} &\equiv [p^1 \cdot q^0 + P^{1L} \cdot Q^0]/[p^0 \cdot q^0 + P^0 \cdot Q^0] \\ &= [p^1 \cdot q^0 + (p^1 \cdot q^0/p^0 \cdot q^0)P^0 \cdot Q^0]/[p^0 \cdot q^0 + P^0 \cdot Q^0] \text{ using (23)} \\ &= (p^1 \cdot q^0/p^0 \cdot q^0)\left[s_q{}^0 + s_Q{}^0\right] \text{ using definitions (10) and (11)} \\ &= P_{Lq}\left[s_q{}^0 + s_Q{}^0\right] \text{ using definitions (8) and (9)} \\ &= P_{Lq}. \end{aligned} \tag{24}$$

Thus the use of P_{Lq} as the indexing index in our inflation adjusted carry forward prices leads to a Laspeyres type index, $P_L{}^{CL}$, that turns out to equal P_{Lq}, the Laspeyres price index for continuing goods and services. This is a useful result since P_{Lq} is a "real" index with no imputations, whereas $P_L{}^{CL}$ is constructed using a great many imputations if N is large. Note that both $P_L{}^C$ and $P_L{}^{CL}$ can be constructed using only knowledge of p^0, p^1, P^0, q^0, q^1 and Q^0.

Comparing (21) with (24) leads to the following inequality:

$$P_L{}^{CL} > P_L{}^C \text{ if and only if } P_{Lq} > 1. \tag{25}$$

Instead of using the Laspeyres price index for continuing commodities, we could use the Paasche index for continuing commodities as the indexing index in the definition of the inflation adjusted carry forward prices for the unavailable commodities. Define the *inflation adjusted carry forward prices*, P^{1P},

22 See for example de Haan and Krsinich (2012, 2014), Diewert et al. (2017) or Diewert (2021c). This approach is implicit in the use of equation (12) in the previous section.

using the Paasche price index for continuing commodities, P_{Pq}, as the inflation adjusting index as follows:

$$P^{1P} \equiv P_{Pq}P^0 = (p^1 \cdot q^1/p^0 \cdot q^1)P^0. \tag{26}$$

Define the Laspeyres index using the inflation adjusted carry forward prices, P^{1P}, in place of the true prices, P^{1*}, as $P_L{}^{CP}$:

$$\begin{aligned} P_L{}^{CP} &\equiv [p^1 \cdot q^0 + P^{1P} \cdot Q^0]/[p^0 \cdot q^0 + P^0 \cdot Q^0] \\ &= [p^1 \cdot q^0 + (p^1 \cdot q^1/p^0 \cdot q^1)P^0 \cdot Q^0]/[p^0 \cdot q^0 + P^0 \cdot Q^0] \text{ using (26)} \\ &= [(p^1 \cdot q^0/p^0 \cdot q^0)p^0 \cdot q^0 + (p^1 \cdot q^1/p^0 \cdot q^1)P^0 \cdot Q^0] \setminus [p^0 \cdot q^0 + P^0 \cdot Q^0] \\ &= (p^1 \cdot q^0/p^0 \cdot q^0)s_q{}^0 + (p^1 \cdot q^1/p^0 \cdot q^1)s_Q{}^0 \text{ using definitions (10) and (11)} \\ &= P_{Lq}s_q{}^0 + P_{Pq}s_Q{}^0 \text{ using definitions (8)} \\ &< P_L{}^{CL} \text{ if and only if } P_{Lq} > P_{Pq}. \end{aligned} \tag{27}$$

Thus, the use of P_{Pq} as the inflation index in our inflation adjusted carry forward prices leads to a price index, $P_L{}^{CP}$, that turns out to equal a share weighted average of P_{Lq} and P_{Pq}, the Laspeyres and Paasche price indexes for continuing goods and services. Because P_{Pq} is likely to be less than its Laspeyres counterpart, P_{Lq}, it is likely that $P_L{}^{CP}$ will be less than $P_L{}^{CL}$. Note that the scope of $P_L{}^{CL}$ and $P_L{}^{CP}$ is the set of all M + N commodities and N imputations are required to construct these indexes, whereas P_{Lq} and P_{Pq} are "real" indexes with no imputations. A knowledge of q^1 is required to construct $P_L{}^{CP}$ whereas no knowledge of q^1 was required to construct $P_L{}^{CL}$.

Using (20) and (24), it is easy to show that $P_L{*} > P_L{}^{CL}$ if and only if $P_{LQ}{*} > P_{Lq}$. Using (20) and (27), it is easy to show that $P_L{*} > P_L{}^{CP}$ if and only if $P_{LQ}{*} > P_{Pq}$. The bottom line is that all three practical indexes that use some form of carry forward pricing for missing products, $P_L{}^{C}$, $P_L{}^{CL}$ and $P_L{}^{CP}$, *will have substantial downward biases* relative to the true Laspeyres index, $P_L{*}$.

It is still unclear whether a given country will in general have rising or falling prices for continuing products as lockdowns take place. Demand for available commodities may fall due to declining household incomes as industries shut down and this may lead to lower prices for continuing products. On the other hand, there may be supply shortages for some highly demanded consumer products like toilet paper, face masks and hand sanitizers, which will lead to higher prices for these products. Workers in supermarkets that are open may demand higher wages to compensate them for increased risk of infection, which will lead to higher grocery prices in general. Truckers will also be subject to increased risk of infection and may get higher wages a result and this will lead to higher prices for available products. Meat processing plants may shut down due to virus spread, which again could lead to higher prices. Also, there will be rapid growth of home deliveries for available products, which will lead to higher prices for products ordered online since transport costs must be included for the home delivered goods. A shortage of delivery trucks and drivers may drive up transportation costs. Border shutdowns may restrict imports of food and medical products, again leading to higher prices for continuing products.

Thus it will be important for statistical agencies to produce inflation estimates for continuing commodities and this can be accomplished if the agency produces the Laspeyres index for continuing products, P_{Lq}. If the agency is able to obtain approximate expenditure data for current period values for continuing products $p^1{\cdot}q^1$, then it would be desirable to produce the Paasche index for continuing product, P_{Pq}, as well so that the Fisher index for continuing products, $P_{Fq} = [P_{Lq}P_{Pq}]^{1/2}$, could also be produced.

4. Constructing a Lowe index when transitioning to a lockdown economy

The analysis in the previous section can be adapted to study the behaviour of *fixed basket Lowe price indexes* when a country closes down industries. This type of index is of interest because it is used in the construction of the CPI in most countries.

Let $q^b \equiv [q_1{}^b,...,q_M{}^b] >> 0_M$ and $Q^b \equiv [Q_1{}^b, ..., Q_N{}^b] >> 0_N$ be a vector of "representative" commodities that households are purchasing in period 0 and prior periods. That is, b denotes the period(s) for which the consumption basket is comprised of these representative commodities. The *fixed basket Lowe price index* going from period 0 to 1, P_B, is defined as follows:

$$\begin{aligned} P_B{}^* &\equiv [p^1 \cdot q^b + P^{1*} \cdot Q^b]/[p^0 \cdot q^b + P^0 \cdot Q^b] \\ &= [(p^1 \cdot q^b/p^0 \cdot q^b)p^0 \cdot q^b + (P^{1*} \cdot Q^b/P^0 \cdot Q^b)P^0 \cdot Q^b]/[p^0 \cdot q^b + P^0 \cdot Q^b] \\ &= P_{Bq}s_q{}^b + P_{BQ}{}^* s_Q{}^b, \end{aligned} \tag{28}$$

where the *fixed basket subindexes* for continuing commodities and unavailable commodities, P_{Bq} and P_{BQ} are defined as follows:

$$P_{Bq} \equiv p^1 \cdot q^b/p^0 \cdot q^b, \tag{29}$$

$$P_{BQ}^* \equiv P^{1*} \cdot Q^b/P^0 \cdot Q^b. \tag{30}$$

The *base period hybrid shares* (prices of period 0 but quantities for a prior year b) for the continuing and disappearing commodity groups, $s_q{}^b$ and $s_Q{}^b$, are defined as follows:

$$s_q{}^b \equiv p^0 \cdot q^b/[p^0 \cdot q^b + P^0 \cdot Q^b] \cdot \tag{31}$$

$$s_Q{}^b \equiv P^0 \cdot Q^b/[p^0 \cdot q^b + P^0 \cdot Q^b]. \tag{32}$$

In definitions (28) and (30), we have used reservation prices to value goods and services that are no longer available. It makes sense to use reservation prices to value unavailable products in the context of the economic approach to index number theory but it is not clear that it is appropriate to use them to value the fixed basket Q^b of unavailable products in period 1. However, the logic of the fixed basket approach works as follows: there is a representative basket of commodities that households are consuming in periods prior to period 1, (q^b, Q^b). The fixed basket methodology simply prices out this basket of goods and services at the prevailing market prices of periods 0 and 1 and the ratio of these costs becomes the fixed basket price index. When goods and services are unavailable in period 1, it is reasonable to use reservation prices for these absent market prices since they are (imputed) *market clearing prices* that will ration demand down to zero for the absent commodities in period 1.

As in the previous section, instead of using reservation prices, the prices P^{1*} could be set equal to the base period prices, giving rise to the following *carry forward basket price index*, $P_B{}^C$, defined as follows:

$$\begin{aligned} P_B{}^C &\equiv \left[p^1 \cdot q^b + P^0 \cdot Q^b\right]/\left[p^0 \cdot q^b + P^0 \cdot Q^b\right] \\ &= \left[(p^1 \cdot q^b/p^0 \cdot q^b)p^0 \cdot q^b + (P^0 \cdot Q^b/P^0 \cdot Q^b)P^0 \cdot Q^b\right]/\left[p^0 \cdot q^b + P^0 \cdot Q^b\right] \\ &= P_{Bq}s_q{}^b + s_Q{}^b \\ &< P_{Bq}s_q{}^b + P_{BQ}{}^*s_Q{}^b \text{ assuming that } P^*_{BQ} \equiv P^{1*} \cdot Q^b/P^0 \cdot Q^b > 1 \\ &= P_B{}^* \text{ using definition (28).} \end{aligned} \tag{33}$$

Hence the carry forward fixed basket index will have a downward bias if $P^{1*}{\cdot}Q^b > P^0{\cdot}Q^b$, which is a reasonable assumption.

An alternative to using carry forward prices is to use the price index for continuing commodities, P_{Bq} defined by (29) to form the *inflation adjusted fixed basket carry forward price vector* $P^{1I} \equiv P_{Bq}P^0 = (p^1{\cdot}q^b/p^0{\cdot}q^b)P^0$ as an estimate for P^{1*}. This leads to the following *inflation adjusted fixed basket price index*, $P_B{}^{CI}$, defined as follows:

$$\begin{aligned} P_B{}^{CI} &\equiv [p^1 \cdot q^b + P^{1I} \cdot Q^b]/[p^0 \cdot q^b + P^0 \cdot Q^b] \\ &= [p^1 \cdot q^b + P_{Bq}P^0 \cdot Q^b]/[p^0 \cdot q^b + P^0 \cdot Q^b] \\ &= [(p^1 \cdot q^b/p^0 \cdot q^b)p^0 \cdot q^b + P_{Bq}(P^0 \cdot Q^b/P^0 \cdot Q^b)P^0 \cdot Q^b]/[p^0 \cdot q^b + P^0 \cdot Q^b] \\ &= P_{Bq}s_q{}^b + P_{Bq}s_Q{}^b \\ &= P_{Bq} \text{ since } s_q{}^b + s_Q{}^b = 1 \\ &< P_{Bq}s_q{}^b + P_{BQ}{}^*s_Q{}^b \text{ assuming that } P_{BQ}{}^* > P_{Bq} \\ &= P_B{}^* \text{ using definition (28).} \end{aligned} \tag{34}$$

Again, it is reasonable to assume that $P_{BQ}^* > P_{Bq}$, i.e., that (imputed) inflation for unavailable commodities is greater than observed inflation for continuing commodities. Thus the version of the fixed basket index that makes use of inflation adjusted carry forward prices for the unavailable commodities, P_B^{CI}, will be less than the "true" fixed basket price index P_B^*.

As was the case in the previous section, we could try to determine which practical fixed basket price index, P_B^C or P_B^{CI}, is greater because bias will be minimized if we choose the maximum of these two indexes. We have:

$$\begin{aligned} P_B^C/P_B^{CI} &= \left[P_{Bq}s_q^b + s_Q^b\right]/P_{Bq} \text{ using definitions (33) and (34)} \\ &= s_q^b + \left[s_Q^b/P_{Bq}\right]. \end{aligned} \tag{35}$$

If $P_{Bq} > 1$, so that fixed basket inflation for continuing commodities is positive, then $P_B^{CI} > P_B^C$ and it is preferable (from the viewpoint of minimizing bias) to use the inflation adjusted carry forward fixed basket price index P_B^{CI}. On the other hand, if $P_{Bq} < 1$, so that there is deflation for continuing products, then it would be preferable to use the carry forward fixed basket price index P_B^C.

The above discussion of the fixed basket price indexes parallels our discussion of the Laspeyres price index. In fact, if $q^b = q^0$ and $Q^b = Q^0$, then the fixed basket Lowe index equals the Laspeyres index. However, in the previous section, the analysis of the Laspeyres index was not the end of the story: in the end, the Laspeyres index was combined with the Paasche index to form an approximation to a cost of living index. *In the present section, there is no Paasche counterpart to the fixed basket index.* A fixed basket index, like the Laspeyres index has a certain amount of *substitution bias*. Normally, if consumption patterns do not change much over time, a fixed basket index that uses a representative basket will not be subject to a great deal of substitution bias. However, in the present context, there are massive changes in the actual consumption vectors as we move from a pre lockdown period to a post lockdown period. And there are massive changes in the corresponding market prices.[23] Thus the amount of substitution bias in a Laspeyres (too high) or Paasche (too low) or fixed basket index (too high) will also be very large indeed under these conditions. When we take an average of the Paasche and Laspeyres indexes to form a Fisher index, we greatly reduce the amount of substitution bias.

In addition to being subject to substitution bias under normal conditions, there is another problem with using a fixed basket price index during a pandemic. The public will regard a fixed basket price index as a "reasonable" index, provided that the basket vectors, q^b and Q^b, are not too far removed from "normal" consumption patterns. A fixed basket index is very easy to

23 We interpret Hicksian reservation prices for unavailable commodities as market prices.

explain and is perfectly reasonable under normal conditions. *But a fixed basket index is not intuitively plausible when a substantial fraction of the fixed basket commodities are simply not available.* In order to maintain public confidence in the CPI, it will be necessary for *statistical agencies to move to more representative baskets that are relevant for lockdown conditions.* This means that the national statistical agency will need to find ways to update their historical baskets more quickly—updated baskets in real time would be ideal.[24]

To explain how updated baskets could work in producing a reasonable CPI for continuing goods and services, suppose the price and quantity data for period 0 are (p^0, q^{0b}) and (P^0, Q^{0b}), but for period 1, the data are (p^1, q^{1b}) and $(P^{1*}, 0_N)$. It is unlikely that national statistical agencies will be able to produce the reservation prices P^{1*} and so we will concentrate on how to construct a price index for continuing commodities. Note that we are assuming that a new representative basket for continuing commodities in period 1, q^{1b}, is available to the agency.[25] Under these assumptions, the following *pseudo Laspeyres Paasche and Fisher price indexes*, P_{Bq0}, P_{Bq1} and P_{BqF}, for continuing commodities could be produced:

$$P_{Bq0} \equiv p^1 \cdot q^{0b}/p^0 \cdot q^{0b} = P_{Bq}, \tag{36}$$

$$P_{Bq1} \equiv p^1 \cdot q^{1b}/p^0 \cdot q^{1b}, \tag{37}$$

$$P_{BqF} \equiv \left[P_{Bq0}P_{Bq1}\right]^{1/2,} \tag{38}$$

where P_{Bq} was defined by (29) and we have used the fact that q^{0b} is equal to our old q^b. The logic for preferring the pseudo Fisher index over its Laspeyres and Paasche counterparts is the usual one: P_{Bq0} and P_{Bq1} are both fixed basket indexes that attempt to measure general inflation going from period 0 to 1. Both indexes are equally plausible so good statistical practice suggests that we take an average of the two to obtain a single point estimate of overall inflation between the periods. If q^{0b} is close to actual period 0 consumption of the continuing commodities, q^0, then P_{Bq0} will be a good approximation to the Laspeyres index, P_{qL}. If q^{1b} is close to actual period 1 consumption of the continuing commodities, q^1, then P_{Bq1} will be a

24 Quantity baskets of the form q^b can be replaced by expenditure share information (the vector of expenditure shares s^b) for the same period provided that price information for the same period, p^b, is also available since ${q_m}^b$ will be proportional to ${s_m}^b/{p_m}^b$ for m = 1,...,M.

25 For some elementary index strata, the statistical agency may have scanner data available. In this case, we set $q^{0b} = q^0$ and $q^{1b} = q^1$; i.e., there is no need to use approximations to q^t in this case. The actual consumption vector for a period is the most representative consumption vector for that period. An elementary index is simply an index constructed at the lowest level of aggregation; see Diewert (2021b).

good approximation to the Paasche index P_{qP}. If both approximations are good, then P_{BqF} will be close to our preferred index, P_{qF}.

If the scope of the lockdown does not change materially during the lockdown, then the statistical agency could go back to using a fixed basket (equal to q^{1b}) for the duration of the lockdown. However, it is unlikely that countries will keep the scope of their lockdowns constant. Thus the number of continuing commodities that are present in two consecutive periods is unlikely to be constant. In this case, constructing chained maximum overlap pseudo Fisher indexes over the lockdown periods is preferable.[26]

If the lockdown ends at the beginning of period τ, then it will be necessary to construct a new basket ($q^{\tau b}$, $Q^{\tau b}$) that is representative for period τ. Recall that the basket for period 0 was (q^{0b},Q^{0b}). In order to construct the period τ price level, instead of comparing period τ prices to the prices of period $\tau - 1$, it is best to compare period τ prices, (p^{τ}, P^{τ}), to the prices of period 0, (p^0,P^0). This is because the overlap of products will be a large between the first period of the end of the lockdown, τ, and the last period before the lockdown, 0. There will be less overlap of products available in non-lockdown period τ with products available during the lockdown periods.[27] Thus it is preferable that the following *pseudo Laspeyres Paasche and Fisher price indexes*, P_{B0}, $P_{B\tau}$ and $P_{B0\tau}$, be produced:

$$P_{B0} \equiv [p^t \cdot q^{0b} + P^t \cdot Q^{0b}]/[p^0 \cdot q^{0b} + P^0 \cdot Q^{0b}], \tag{39}$$

$$P_{Bt} \equiv [p^t \cdot q^{tb} + P^t \cdot Q^{tb}]/[p^0 \cdot q^{tb} + P^0 \cdot Q^{tb}], \tag{40}$$

$$P_{B0t} \equiv [P_{B0}P_{Bt}]^{1/2}. \tag{41}$$

Thus the price level in period τ will be set equal to the price level in period 0 times the pseudo Fisher index, $P_{Bq0\tau}$.[28] This index will approximate the true Fisher index going from period 0 to period τ. However, the sequence of price levels going from period 1 to period $\tau - 1$ will not approximate economic price indexes or cost of living indexes because the big increase in the cost of living due to the lockdown in period 1 will not be reflected by the

26 There may be a chain drift problem at the lowest level of aggregation. In this case, it may be necessary to use a multilateral index number. See Ivancic et al. (2011), Diewert and Fox (2022) and Diewert (2021b) for discussions of the chain drift problem and the use of multilateral methods. At higher levels of aggregation, chain drift will typically not be a major problem.

27 Other forms of linking based on the similarity of the structure of prices and quantities going from one period to another period. For an explanation of how to implement these more sophisticated methods of linking observations over time, see section 20 of Diewert (2021c).

28 Pseudo Fisher price indexes have been computed retrospectively; e.g., see Diewert et al. (2009).

movements in the above basket price indexes for periods 1 to $\tau - 1$. In order to accurately measure the cost of living for those periods, we need estimates for the period 1 reservation prices P^{1^*}.

5. Conclusion

The current pandemic conditions have never been experienced since the establishment of modern economic statistics, hence responses to ensure the continued production of high quality economic statistics on key variables in these conditions have not previously been rigorously laid out.

The current recommended approaches for dealing with products disappearing due to lockdowns or consumers stockpiling draw on the standard response to disappearing products in any period. That is, the advice is essentially to continue with current practice as if nothing has happened, using imputed prices for missing goods and expenditure weights from a pre-lockdown period. For example, Eurostat advice for construction of the Harmonized Index of Consumer Prices (HICP) includes the following:[29]

1. "The HICP weights are updated at the beginning of each year and are kept constant throughout the year. Thus, the weights will not change this year as a result of the impact of the COVID-19 on expenditures."
2. All sub-indices for the product classifications "will be compiled even when for some categories no products are available on the market."

We have demonstrated that following the advice on price imputations will lead to overstated estimates of changes in real consumption and understated estimates of changes in the cost of living. We have pointed out the problems of using expenditure weights that are irrelevant for the periods under consideration.[30] Steps that can be taken by national statistical offices to mitigate these problems during a time of lockdown are:

1. Collection of all available prices, including from non-traditional sources. Use inflation adjusted carry forward prices in the case of missing prices; while we favour using reservation prices, we acknowledge that currently it is unlikely that NSOs will be able to estimate these in a timely fashion.

29 The HICP is the European Union's standardized measure of consumer price inflation that each member country must construct. See technical appendix C for an extended quote of the advice, along with information on the recommendations from the International Monetary Fund, UNECE and US Bureau of Labor Statistics.

30 See Diewert and Fox (2020) for consideration of a range of other difficult measurement problems that arise.

2. Dedicate resources to the collection of current expenditure weights for the consumption basket.
3. Produce an analytical CPI that can be revised as new methodology is developed and new data sources become available.[31]

As a general longer-term goal, statistical agencies need to move away from a fixed basket Lowe index and attempt to produce approximations to Laspeyres and Paasche indexes, and hence Fisher indexes. This will allow consumer price inflation to be measured by an index that is relevant to current consumer expenditure patterns.[32]

It is unlikely that expenditure patterns will return to those observed before the first lockdown period, making it doubtful that inflation adjusted carry forward pricing will produce an accurate index when the lockdown period ends. But what is clear is that the use of adjusted carry forward prices will not produce an accurate CPI within the lockdown period. New basket information is required in order to produce a meaningful CPI within the lockdown period. Thus establishing a continuous consumer expenditure survey is key to producing a meaningful CPI during these turbulent times.

Supporting information

Supplementary material accompanies this article.

References

Australian Bureau of Statistics (2017) “An Implementation Plan to Maximise the Use of Transactions Data in the CPI,” Information Paper 6401.0.60.004, 16 June, Canberra

Carvalho, V.M., J.R. Garcia, S. Hansen, Á. Ortiz, T. Rodrigo, J.V. Rodríguez Mora, and P. Ruiz (2021) “Tracking the COVID-19 crisis with high-resolution transaction data,” *Royal Society Open Science* 8(8), 210218. `https://doi.org/10.1098/rsos.210218`

Cavallo, A. (2020) “Inflation with Covid consumption baskets,” NBER working paper no.27352. Available at `www.nber.org/papers/w27352`

de Haan, J., and F., Krsinich (2012) “The treatment of unmatched items in rolling year GEKS price indexes: Evidence from New Zealand scanner data,” paper

31 The US Bureau of Labor Statistics (BLS) approach to dealing with the coronavirus pandemic is very much in line with the approach advocated in this paper. The example set by the BLS shows that it is not impossible to produce household expenditure information in real time. See technical appendix A for more on their recommendations.

32 The production of approximate Walsh or Törnqvist indexes as alternatives to the Fisher index is also possible. Note that the BLS produces an approximation to the Törnqvist index in real time.

presented at the Economic Measurement Workshop, November 23, 2012, University of New South Wales
—— (2014) "Scanner data and the treatment of quality change in nonrevisable price indexes," *Journal of Business & Economic Statistics* 32(3), 341–58. https://doi.org/10.1080/07350015.2014.880059
Diewert, W.E. (1976) "Exact and superlative index numbers," *Journal of Econometrics* 4, 115–45. https://doi.org/10.1016/0304-4076(76)90009-9
—— (1992) "Fisher ideal output, input and productivity indexes revisited," *Journal of Productivity Analysis* 3, 211–48
—— (1998) "Index number issues in the consumer price index," *Journal of Economic Perspectives* 12(1), 47–58
—— (2021a) "The economic approach to index number theory." Chapter 5 (draft), *Consumer Price Index Theory*. Washington: International Monetary Fund
—— (2021b) "Elementary indexes." Chapter 6 (draft), *Consumer Price Index Theory*. Washington: International Monetary Fund
—— (2021c) "The chain drift problem and multilateral indexes," Chapter 7, *Consumer Price Index Theory*. Washington: International Monetary Fund
Diewert, W.E., and K.J. Fox (2020) "Measuring real consumption and CPI bias under lockdown conditions," NBER working paper no. 27144. Available at www.nber.org/papers/w27144
—— (2022) "Substitution bias in multilateral methods for CPI construction," *Journal of Business & Economic Statistics* 40(1), 335–69. https://doi.org/10.1080/07350015.2020.1816176
Diewert, W.E., K.J. Fox, and P. Schreyer (2017) "The digital economy, new products and consumer welfare," discussion paper 17–09, Vancouver School of Economics, Vancouver School of Economics discussion paper no. 17–09
Diewert, W.E., M. Huwiler, and U. Kohli (2009) "Retrospective price indices and substitution bias," *Swiss Journal of Economics and Statistics* 145(20), 127–35
Dunn, A., K. Hood, and A. Driessen (2020) "Measuring the effects of the COVID-19 pandemic on consumer spending using card transaction data," U.S. Bureau of Economic Analysis working paper no. WP2020-5
Eurostat (2020) "Guidance on the compilation of the HICP in the context of the COVID-19 crisis," methodological note. Available at https://ec.europa.eu/eurostat/documents/10186/10693286/HICP_guidance.pdf
Hausman, J.A. (1996) "Valuation of new goods under perfect and imperfect competition." In T.F. Bresnahan and R.J. Gordon, eds., *The Economics of New Goods*, pp. 207–48. Chicago: University of Chicago Press
—— (1999) "Cellular telephone, new products and the CPI," *Journal of Business & Economic Statistics* 17(2), 188–94
—— (2003) "Sources of bias and solutions to bias in the consumer price index," *Journal of Economic Perspectives* 17(1), 23–44
Hicks, J.R. (1940) "The valuation of the social income," *Economica* 7, 105–24
International Monetary Fund (2020) "Consumer price index continuity," Unit C-4, Price Statistics, April. Available at https://statswiki.unece.org/display/CCD2/Compilation+of+CPI+in+times+of+COVID-19?preview=/278037166/279773977/STA%20CPI%20Business%20Continuity.pdf
Ivancic, L., W.E. Diewert, and K.J. Fox (2011) "Scanner data, time aggregation and the construction of price indexes," *Journal of Econometrics* 161, 24–35. https://doi.org/10.1016/j.jeconom.2010.09.003

Konüs, A.A. (1924) “The problem of the true index of the cost of living,” [translation] *Econometrica* 7(1), 10–29

Konüs, A.A., and S.S., Byushgens (1926) “K probleme pokupatelnoi cili deneg,” *Voprosi Konyunkturi* 7, 151–72

Triplett, J. (2004), *Handbook on Hedonic Indexes and Quality Adjustments in Price Indexes*: *Special Application to Information Technology Products.* Paris: OECD Publishing. https://doi.org/10.1787/9789264028159-en

UNECE (2020) “Compilation of CPI in Times of COVID-19.” Available at https://statswiki.unece.org/display/CCD2/Compilation+of+CPI+in+times+of+COVID-19

von Hofsten, E. (1952) *Price Indexes and Quality Change*. London: George Allen and Unwin